

國立臺中教育大學 103 學年度大學日間部轉學招生考試

微積分試題

適用學系：數學教育學系二年級

注意事項：請將答案寫在答案卷上，並請標明題號。

一、填充題（50%，每格 5%）

1. 試求 $\int \frac{1}{x \ln x} dx = \underline{\hspace{2cm}}$.

2. 試求 $\int_0^2 \frac{1}{\sqrt[3]{x-1}} dx = \underline{\hspace{2cm}}$.

3. 試求 $\frac{d}{dx} \left[\left(\frac{x-3}{x^2+1} \right)^2 \right] = \underline{\hspace{2cm}}$.

4. Evaluate $\lim_{x \rightarrow \infty} (xe^{1/x} - x) = \underline{\hspace{2cm}}$.

5. Evaluate $\int \frac{2x^2 - x + 4}{x^3 + 4x} dx = \underline{\hspace{2cm}}$.

6. The average value of the function $f(x) = \cos^4 x \sin x$ on the interval $[0, \pi]$ is $\underline{\hspace{2cm}}$.

7. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n} = \underline{\hspace{2cm}}$.

8. Let $y = f\left(\frac{2x-1}{x+1}\right)$ and $f'(x) = \cos x$. Then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

9. Evaluate $\int_0^{\infty} x^3 e^{-x} dx = \underline{\hspace{2cm}}$.

10. The Taylor's series representation of $f(x) = \frac{1}{x}$ at $x=1$ is $\underline{\hspace{2cm}}$.

【背面尚有試題】

二、計算及證明題 (50%，每題 10%)

1. 若 u 是 x 的可微函數，且 $a > 0$ ，請證明 $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
2. A number a is called a fixed point of a function f if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x , then f has at most one fixed point.
3. Find the extreme values of $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.
4. Evaluate the integral $\int_0^1 \int_{2x}^2 \int_0^1 z^3 e^{y^2} dz dy dx$.
5. Find the area of the region which is outside the graph of the polar equation $r = 2$ and inside the graph of the equation $r^2 = 8 \sin 2\theta$.

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微積分試題

適用學系：數學教育學系三年級

注意事項：請將答案寫在答案卷上，並請標明題號。

一、填充題（50%，每格 5%）

1. 試求 $\int \arctan x dx = \underline{\hspace{2cm}}$.
2. 已知 $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0$ ，求 $\frac{\partial z}{\partial x} = \underline{\hspace{2cm}}$.
3. 試求 $\int_0^{\infty} \frac{1}{e^x + e^{-x}} dx = \underline{\hspace{2cm}}$.
4. Evaluate $\lim_{x \rightarrow 3} \left(\frac{x}{x-3} \int_3^x \frac{\sin t}{t} dt \right) = \underline{\hspace{2cm}}$.
5. If $x^y = y^x$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
6. Evaluate the indefinite integral $\int x^5 \sqrt{1+x^2} dx = \underline{\hspace{2cm}}$.
7. The area enclosed by the line $x = y$ and the parabola $4x + y^2 = 12$ is $\underline{\hspace{2cm}}$.
8. Let $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x + 1} - ax - b) = 0$. Then $b = \underline{\hspace{2cm}}$.
9. Let $f(x, y) = x^{xy}$. Then $\frac{\partial f}{\partial x} = \underline{\hspace{2cm}}$.
10. The Maclaurin's series for $f(x) = \ln(1+x)$ is $\underline{\hspace{2cm}}$.

【背面尚有試題】

二、計算及證明題 (50%，每題 10%)

1. 將圓面 $x^2 + y^2 \leq 1$ 繞直線 $x = 2$ 旋轉一圈，試求所得旋轉體的體積。

2. (a) Prove that if f is a continuous function, then $\int_0^a f(x) dx = \int_0^a f(a-x) dx$.

(b) Use part (a) to show that $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4}$ for all positive numbers n .

3. Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

4. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(x^2 + y^2) dy dx$.

5. Evaluate $\int_0^{\pi/4} \tan^{2n} x dx$.