國立臺中教育大學 101 學年度大學日間部轉學招生考試

微積分試題

適用學系:數學教育學系二年級

（本試題共二頁）

一、填充題（40%，每格 5%）

1. 求 $\displaystyle\int_1^\infty x^2 \ln x\,dx$ = _____.

2. 若 $x\sin y = y\cos x$，求 $\dfrac{dy}{dx} =$ _____.

3. Evaluate $\displaystyle\lim_{x\to 1^+}\left(\dfrac{x}{x-1}-\dfrac{1}{\ln x}\right)=$ _____.

4. An equation of the tangent line to the curve $x^2+xy+y^2=3$ at the point $(1,1)$ is _____.

5. The directional derivative of the function $f(x,y)=x^2y^3-4y$ at the point $(2,-1)$ in the direction of the vector $\mathbf{v}=\langle 2,5\rangle$ is _____.

6. Evaluate $\displaystyle\lim_{x\to 1}\dfrac{\displaystyle\int_1^x e^t\,dt}{x-1}=$ _____.

7. Calculate $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\dfrac{1}{1+\cos x}\,dx=$ _____.

8. Calculate $\displaystyle\int_{-2}^{2}\left|x^2-x-2\right|\,dx=$ _____.

二、計算及證明題（60%，每題 10%）

1. 試求 $h(x)=\dfrac{2x+3}{x-4}$ 函數圖形的鉛直漸近線和水平漸近線。

2. 試求 $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ 繞 y 軸旋轉後所得旋轉體的體積。

3. Evaluate the integral $\displaystyle\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$.

4. Let $f(x) = \begin{cases} |x|^x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$.

  (a) Show that f is continuous at 0.

  (b) Show that f is not differentiable at 0.

5. Find the area of the region bounded by the graphs of $y = |x^2 - 1|$ and $y = 2$.

6. Find the radius of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \dfrac{(nx)^n}{n!}$.

國立臺中教育大學 101 學年度大學日間部轉學招生考試

微積分試題

適用學系:數學教育學系三年級

（本試題共二頁）

一、填充題（52%，每格 4%）

1. 試求 $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = $ _____.

2. 試求 $\displaystyle\int_0^{\pi/2}\cos^4 x\,dx = $ _____.

3. 試求 $\displaystyle\int_0^{\infty}\frac{1}{\sqrt{x}(x+1)}dx = $ _____.

4. If $\displaystyle\lim_{x\to a}\big[f(x)+g(x)\big]=2$ and $\displaystyle\lim_{x\to a}\big[f(x)-g(x)\big]=1$, then $\displaystyle\lim_{x\to a}f(x)g(x) = $ _____.

5. If $\displaystyle F(x)=\int_1^x f(t)dt$, where $\displaystyle f(t)=\int_1^{t^2}\frac{\sqrt{1+u^4}}{u}du$, find $F''(2) = $ _____.

6. Suppose f is a one-to-one differentiable function and its inverse function f^{-1} is also differentiable. If $f(4)=5$ and $f'(4)=\dfrac{2}{3}$, find $(f^{-1})'(5) = $ _____.

7. The Maclaurin series for the function $f(x)=x\cos x$ is _____.

8. Evaluate the integral $\displaystyle\int_0^1\int_{3y}^3 e^{x^2}dxdy = $ _____. (Hint: change the order of integration)

9. Let $x^2\cos y = y^2\sin x$. Then $\dfrac{dy}{dx} = $ _____ .

10. Calculate $\displaystyle\lim_{n\to\infty}\frac{1^5+2^5+3^5+\cdots+n^5}{n^6} = $ _____ .

11. Calculate $\int_1^\infty \dfrac{1}{x(1+x^2)}\,dx = $_____ .

12. Let $\int_0^{x^2} tf(t)\,dt = x^2 + x^4$. Then $f(1) = $_____ .

13. Evaluate $\int_3^5 \dfrac{1}{\sqrt{x^2-9}}\,dx = $_____ .

二、計算及證明題（48%，每題 8%）

1. 試求區間 $[0,1]$ 上的圖形 $f(x) = x^3$ 繞 x 軸旋轉所得旋轉體的面積。

2. 試求圖形 $y = 4 - x^2$ 上哪一點距離 $(0,2)$ 最近？

3. If $C_0 + \dfrac{C_1}{2} + \cdots + \dfrac{C_{n-1}}{n} + \dfrac{C_n}{n+1} = 0$, where $C_0, \cdots, C_n$ are real constants, prove that the equation $C_0 + C_1 x + \cdots C_{n-1} x^{n-1} + C_n x^n = 0$ has at least one real root between 0 and 1.

4. If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rational x, prove that f is not Riemann integrable on $[a,b]$ for any $a < b$.

5. Prove that $\displaystyle\lim_{(x,y)\to(0,0)} \dfrac{x-y}{x+2y}$ does not exist.

6. Evaluate the double integral $\int_0^{\frac{\pi}{2}} \int_y^{\frac{\pi}{2}} \dfrac{\sin x}{4x}\,dx\,dy$.