國立臺中教育大學 111 學年度學士班日間部轉學生招生考試

微積分試題

【本考科得以鉛筆作答】

適用學系：數學教育學系三年級

一、填充題（每題 4%，共 84%）

1. Find $\lim_{x\to 3}\dfrac{x^2-x-6}{x-3}=$_____.

2. Calculate $\int_0^{\frac{\pi}{4}} 1+\sqrt{1+\tan^2 x}\,dx=$_____.

3. Find $\int_{3/2}^{9/4} \dfrac{1}{\sqrt{3x-x^2}}\,dx=$_____.

4. Evaluate $\int_0^{\frac{\pi}{2}} \cos^4 x\,dx=$_____.

5. Calculate $\int_0^2 |2x-1|\,dx=$_____.

6. Let $f(x)=\dfrac{1}{4}x^3+x$. Then $\left(f^{-1}\right)'(4)=$_____.

7. Calculate $\int_0^1 \tan^{-1} x\,dx=$_____.

8. Evaluate $\lim_{x\to 1^+}\left(\ln(x^7-1)-\ln(x^2-1)\right)$._____.

9. The curves $y=x^2+ax+b$ and $y=cx-x^2$ have a common tangent line at the point $(1,0)$. Find the constants a, b, and c.=_____.

10. If $x^{\sin y}=\ln y$, find $\dfrac{dy}{dx}$.=_____.

11. Find equations for all horizontal and vertical asymptotes for the graph of $y=\frac{\sqrt{2x^2+4}}{x-3}$. =_____.

12. At which points on the curve $y=-3x^5-5x^4+20x^3+60x$ does the tangent line have the largest slope? = _____.

13. Find the area of the region under the curve $y=\frac{x^2+1}{3x-x^2}$ from $x=1$ to $x=2$. =_____.

14. Evaluate $\int \sin^{-1} x\,dx$. =_____.

15. 求$\lim_{x\to 0}\frac{2-2\cos x}{\sin x}$= _____.

16. 求$\lim_{x\to\infty}(lnx)^{2/x}$= _____.

17. 求級數$\sum_{n=0}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)$的和？_____.

18. 求$\int_0^{\pi} x\sin x\,dx$ =_____.

19. 求$\int_1^{\infty}\frac{lnx}{x^2}dx$ =_____.

20. 求$\int_{-\pi/2}^{\pi/2} cos^3 x dx$ = _____.

21. 求$\int_1^{\infty}\int_0^{1/x} ydydx$=_____.

二、證明題（每題 8%，共 16%）

1. Let $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0),\\ 0 & \text{if } (x,y)=(0,0).\end{cases}$

   Show that $f_x(0,0)$ and $f_y(0,0)$ both exist but f is not differentiable at $(0,0)$.

2. Find the volume of the solid generated when the region under the curve $\mathrm{y}=x^2$ over the interval $[0,2]$ is rotated about the line $\mathrm{y}=-1$.