

微積分試題

適用學系：數學教育學系二、三年級

【注意事項：1.請將答案寫在答案卷上，並標示題號 2.本科目得以鉛筆作答】

一、填充題(20%)

1. 求 $\lim_{x \rightarrow 0} \frac{\tan x}{x} = (\quad) (5\%)$
2. 求 $\int_0^{\infty} \frac{dx}{\sqrt{x}(x+1)} = (\quad) (5\%)$
3. 求以圖形 $y = x^3 + x + 1$ 、 $y = 1$ ，和 $x = 1$ 為界的區域，繞直線 $x = 2$ 旋轉所得旋轉體的體積？ $(\quad) (5\%)$
4. 正弦曲線與餘弦曲線相交無窮多次，圍出的區域面積都相等，請求出單一個圍出區域的面積？ $(\quad) (5\%)$

二、計算證明題(80%)

1. Find numbers a and b such that $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-2}{x} = 1. (10\%)$
2. Let $f(x) = \begin{cases} \sqrt[3]{x^4} \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$
 - (a) Is $f(x)$ continuous at $x = 0$? (5%)
 - (b) Compute $f'(x)$ for $x \neq 0$ and $f'(0)$. (5%)
3. Find the average value of $f(x) = \sqrt{4-9x^2}$, $0 \leq x \leq \frac{2}{3}$. (10%)

4. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Please state the tests which you use.

(a) $\sum_{n=1}^{\infty} \frac{\sin(n\pi/6)}{1+n\sqrt{n}}$ (5%)

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ (5%)

5. Find the area of the region enclosed by $y = x^2 + 3$ and $y = -x^2 + 2x + 3$. (10%)

6. Calculate $\int_0^2 \ln x \, dx$. (10%)

7. Evaluate the iterated integral $\int_0^1 \int_x^1 2 \sin(y^2) \, dy dx$. (10%)

8. Find the local maximum and minimum values and saddle points of

$f(x, y) = x^4 + y^4 - 4xy + 8$. (10%)