國立臺中教育大學 102 學年度大學日間部轉學招生考試

微積分 試題

適用學系：數學教育學系　　二年級

（本試題共二頁）

一、填充題（50%，每格 5%）

1. 試求 $\int \sec^3 x dx =$__________.

2. 試求 $\frac{d}{dx}\left[2^{3x}\right] =$ __________.

3. 試求 $\lim_{x\to 0^+}(\sin x)^x =$ __________.

4. Calculate $\int_0^1 x\sqrt{5x+4}\, dx =$__________ .

5. Calculate $\lim_{n\to\infty}\left(\frac{(2n)!}{n!n^n}\right)^{\frac{1}{n}} =$__________ .

6. Evaluate $\lim_{(x,y)\to(0,0)}\frac{x^3+y^3}{x^2+y^2} =$__________ .

7. Let $f(x) = 2x + \cos x$. Find $(f^{-1})''(1) =$ _________.

8. Let $f(x) = \int_0^{\sqrt{x}} \frac{1}{\sqrt{1+t^4}}\, dt$. Find $f'(1) =$ _________.

9. Evaluate $\lim_{x\to\infty}\left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x =$ _________.

10. Let M and m be the absolute maximum and absolute minimum values of $f(x) = (x^2-1)^3$ on the interval $[-1, 2]$ respectively. Find $M + m =$ _________.

【背面尚有試題】

二、計算及證明題（50%，每題 10%）

1. 請描述並且證明均值定理(Mean Value Theorem)。

2. Let $z = f(u^2 + v^2)$. Prove $u\frac{\partial z}{\partial v} = v\frac{\partial z}{\partial u}$.

3. Prove $\lim_{x \to 2}\left(\frac{1}{2}x + 2\right) = 3$ by using the definition of ε and δ.

4. Let $f(x, y) = 3x - x^3 - 2y^2 + y^4$. Find the critical points of $f(x, y)$ and classify them (i.e., determine whether the critical points are points of local maximum, local minimum values or saddle points).

5. Show that the volume of a pyramid with square base of side L and height h is $\frac{1}{3}L^2 h$.

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微積分 試題

適用學系:數學教育學系　　三年級

（本試題共二頁）

一、填充題（50%，每格 5%）

1. 試求 $\int e^{2x}\sin 3x dx=$ ＿＿＿＿＿＿.

2. 試求 $\lim_{x\to 1^+}(x-1)^{\ln x}=$ ＿＿＿＿＿＿.

3. 試求 $\sum_{n=0}^{\infty}\left[\left(\frac{2}{3}\right)^n-\frac{1}{(n+1)(n+2)}\right]=$ ＿＿＿＿＿＿.

4. Let $E=\bigcup_{n=1}^{\infty}(-n,n)$. Then the boundary of E is ＿＿＿＿＿ .

5. Calculate $\lim_{n\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{n\pi}{n}\right)=$ ＿＿＿＿＿.

6. Evaluate $\int_0^{\infty}\frac{e^x}{e^{2x}+1}dx=$ ＿＿＿＿＿ .

7. Suppose that a is a real number such that the limit $\lim_{x\to -2}\frac{3x^2+ax+a+3}{x^2+x-2}=b$ exists. Find $(a,b)=$ ＿＿＿＿.

8. Let L be the normal line to the parabola $y=x^2-5x+4$ that is parallel to the line $x-3y=5$. Then the equation of L is ＿＿＿＿.

9. Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sin^2\frac{k\pi}{n}=$ ＿＿＿＿.

10. The volume of the solid that lies under the paraboloid $z=x^2+y^2$ and above the region D in the xy-plane bounded by the line $y=2x$ and the parabola $y=x^2$ is ＿＿＿＿.

【背面尚有試題】

二、計算及證明題（50%，每題 10%）

1. 請證明 $\int \tan^n x dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x dx$

2. Find the maximum value of the directional derivative of $f(x, y) = 3x^2 y + 4x$ at the point $(1,1)$.

3. Let $\{x_n\}$ be a sequence of real numbers. Show that if $\{x_n\}$ is convergent, then $\{x_n\}$ is bounded.

4. Let f be continuous on $[0,\pi]$. Show that $\int_0^{\pi} xf(\sin x)\, dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$.

5. Let $f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2} & \text{if } (x, y) \neq (0,0) \\ 0 & \text{if } (x, y) = (0,0) \end{cases}$. Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.