

國立臺中教育大學 108 學年度學士班日間部轉學生招生考試

微積分試題

【本考科得以鉛筆作答】

適用學系：數學教育學系二、三年級

一、填充題（每題 5%，共 75%）

1. 求 $\lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x =$ _____

2. 請問級數 $\sum_{n=0}^{\infty} \frac{1}{n!}$ ，是否收斂？_____

3. 求 $\int_{-1}^2 2^x dx$ _____

4. 求 $\int_0^3 \frac{x^3}{\sqrt{x^2+9}} dx$ _____

5. 求右列兩函數圍出區域圖形的面積： $f(x) = \sqrt[3]{x-1}$ 、 $g(x) = x-1$ ？_____

6. 求以右列函數圖形為界的區域： $y = 2x - x^2, y = 0$ ，繞直線 $x = 4$ 旋轉，所得旋轉體的體積？_____

7. Let $g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$. Find the constant c that makes g continuous on $(-\infty, \infty)$. $c =$ _____.

（背面尚有試題）

8. Let $y = f(u)$ and $u = g(x)$, where f and g are twice differentiable functions with $f(1) = -1$, $f'(1) = -2$, $f''(1) = 3$, $g(0) = 1$, $g'(0) = 2$, and $g''(0) = 3$. Then $\left. \frac{d^2 y}{dx^2} \right|_{x=0} =$ _____.
9. Let $f(x) = \int_0^{g(x)} \frac{1}{\sqrt{1+t^3}} dt$, where $g(x) = \int_0^{\cos x} (1 + \sin(t^2)) dt$.
Find $f'\left(\frac{\pi}{2}\right) =$ _____.
10. Given that $f(x) = \frac{\ln x}{x}$, the maximum value of $f(x)$ is _____.
11. Evaluate $\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) =$ _____.
12. Find the inverse function $f^{-1}(x)$ of $f(x) = \frac{4x-1}{2x+3}$. $f^{-1}(x) =$ _____.
13. Find the area of the region bounded by the curves $y = \sin^2 x$, $y = \sin^3 x$, $0 \leq x \leq \pi$. _____.
14. Find the directions in which the directional derivative of $f(x, y) = x^2 + \sin xy$ at the point $(1, 0)$ has the value 1. _____.
15. The area under the curve $y = \sin \sqrt{x}$ from $x = 0$ to $x = \pi^2$ is _____.

二、計算證明題（請列出計算過程或畫圖證明，每題 5%，共 25%）

1. 求以拋物面 $z = 4 - x^2 - 2y^2$ ，和 xy - 平面為界的立體區域體積。

2. If $\lim_{x \rightarrow \infty} \left(\frac{x+a}{x-a} \right)^x = e$ ，then $a = ?$

3. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n3^n}$.

（背面尚有試題）

4. Find the maximum positive number $\delta = \underline{\hspace{2cm}}$ such that: If $0 < |x| < \delta$ then $|e^{-x} - 1| < \frac{1}{2}$.

5. Evaluate $\int \ln(x^2 + 1) dx = ?$