國立臺中教育大學 113 學年度日間部學士班轉學生招生考試

微積分試題

【①本考科得使用鉛筆；②並請一律於答案卷上作答】

適用學系：數學教育學系二、三年級

一、 填充題（每題 5%，共 70%）

1. Evaluate $\displaystyle\lim_{x \to 2^-} \frac{|2x-4|}{x-2} =$ _____ .

2. Evaluate $\displaystyle\lim_{x \to 0} \frac{\sqrt{x+1}-1}{2x} =$ _____ .

3. Find $\dfrac{d}{dx}\left[\dfrac{\cos x}{x} \right] =$ _____ .

4. Evaluate $\dfrac{d}{dx}\left[\ln(\ln x^2) \right] =$ _____ .

5. Find $\dfrac{d}{dx}\left[\displaystyle\int_1^{x^2} \dfrac{1}{t}\,dt \right] =$ _____ .

6. Evaluate $\displaystyle\int_0^2 |4x-2|\,dx =$ _____ .

7. Evaluate $\displaystyle\int_2^6 \frac{2x}{\sqrt{x-2}}\,dx =$ _____ .

（背面尚有試題）

8. Evaluate $\displaystyle\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx =$ _____.

9. Find $\displaystyle\int_1^2 \int_0^{\ln x} 4x\,dy\,dx =$ _____.

10. The slope of the tangent line to the curve $f(x) = x + \dfrac{4}{x}$ at the point $(-4, -5)$ is _____.

11. The area of the region enclosed by $x = y^2$ and $y = x - 2$ is _____.

12. Let $f(x, y) = 3xe^{x^2 y}$. Then $f_y(1, \ln 2) =$ _____.

13. The directional derivative of $f(x, y, z) = x^2 y - yz^3 + z$ at the point $(1, -2, 0)$ in the direction of the vector $(2, 1, -2)$ is _____.

14. If $y^3 + y^2 + 5y - x^2 - 1999 = 0$, then $\dfrac{dy}{dx} =$ _____.

二、計算證明題（**需書寫詳細計算證明過程，每題 10%，共 30%**）

1. A winch at the top of a 12-meter building pulls a pipe of the same length to a vertical position, as shown in the figure. The winch pulls in rope at a rate of -0.2 meter per second. Find the rate of vertical change and the rate of horizontal change at the end of the pipe when $y = 6$ meters.
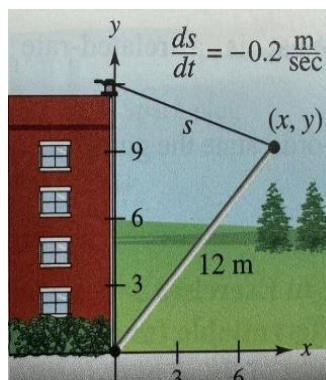
2. Find the general solution of the differential equation $y \ln x - xy' = 0$.

3. Find the volume of the solid generated by revolving the plane region bounded by the graphs of $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.