准考證號碼：

# 國立臺中教育大學 105 學年度學士班日間部轉學招生考試

# 微積分試題

適用學系：數學教育學系二、三年級

考生注意事項：

1. 請考生於答案卷非選擇題及非是非題作答區填答並標示題號。
2. 本考科限用黑色或藍色之原子筆或鋼筆作答。

## 一、填充題（50%，每格 5%）

1. Suppose that $f(x)$ is a function that satisfies the equation $f(s+t)=f(s)+f(t)+s^2t+st^2$ for all real numbers $s$ and $t$. Suppose also that $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=1$. Then $f'(x)=$ ________.

2. Find the interval $I$ on which the curve $y=\int_0^x \dfrac{1}{1+t+t^2}\,dt$ is concave upward. $I=$ ________.

3. Let $R=\{(x,y)\,|\,1\le x\le 3,\ 2\le y\le 5\}$, and let $[\![x]\!]$ denote the greatest integer function. The integral $\iint\limits_R [\![x+y]\!]\,dA=$ ________.

4. Let $x^2+y^2=17$. Then $\dfrac{d^2y}{dx^2}=$ ________.

5. Let $f(x)=\dfrac{\sqrt{2x^2-81}}{3x+15}$. Then the horizontal asymptotes of the graph of $f$ are ________.

6. The directional derivative of $f(x,y,z)=x\sin(yz)$ at $(1,3,0)$ in the direction of $v=(1,2,3)$ is ________.

7. Calculate $\int_0^1 \dfrac{2x}{x^2+2x+1}\,dx=$ ________.

8. Evaluate $\int_0^{\infty} xe^{-2x}dx =$ ________.

9. Evaluate $\lim\limits_{x\to\frac{\pi}{2}^-}(x-\frac{\pi}{2})\tan x =$ ________.

10. Evaluate $\sum\limits_{k=1}^{\infty} 5(\frac{-2}{3})^{2k-1} =$ ________.

**二、計算及證明題（50%，每題 10%）**

1. Find a function $f$ such that $f'(-1)=\frac{1}{2}$, $f'(0)=0$, and $f''(x)>0$ for all $x\in\mathbb{R}$, or prove that such a function cannot exist.

2. Let $f(x)=\sum\limits_{n=1}^{\infty}\frac{x^n}{n^2}$. Find the intervals of convergence for $f(x)$, $f'(x)$, and $f''(x)$.

3. Find the length of the arc of $y=\frac{1}{6}x^3+\frac{1}{2x}$ from $x=1$ to $x=2$.

4. Find the area of the region $D$ bounded above by the line $y=x$ and below by the circle $x^2+y^2-2y=0$.

5. If $u=\frac{1}{2}(x^2+y^2)$ and $v=\frac{1}{2}(x^2-y^2)$, with $x>0,\ y>0$. Please express the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ in terms of $u$ and $v$.