國立臺中教育大學99學年度大學日間部轉學招生考試

微積分試題

適用學系：數學教育學系二年級

一、填充題（40％，每格5％）

1. The derivative of f at x is given by $f'(x)=\lim\limits_{h\to 0}$ ________. Equivalently, $f'(x)=\lim\limits_{t\to x}$ ________.

2. The slope of the tangent line L to the graph of $y=x^2$ at the point $(1,1)$ is ________. Moreover, the equation of L is ________.

3. Evaluate $\int_0^1 xe^{-x^2}dx$ =__________.

4. Evaluate $\lim\limits_{h\to 0}\dfrac{\ln(1+h)}{h}=$ __________.

5. Let $f(x)=\begin{cases} x^2+a & \text{if } x>-3 \\ 8 & \text{if } x=-3 \\ mx+b & \text{if } x<-3 \end{cases}$. Find the values of a, m, b that make f differentiable everywhere. $(a,m,b)=$ ________.

6. The length of the curve $y=1+6x^{3/2}, 0\le x\le 1$ is ________.

二、計算及證明題（60％，每題10％）

1. Evaluate $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^3}$.

2. Find $\lim\limits_{x\to 0}\dfrac{1}{x}\int_0^x \dfrac{1+t}{2+t}dt$.

3. Find the limit or explain that it does not exist $\lim_{x\to\infty} |x|^x$.

4. Prove that if $\lim_{x\to c} f(x) = A$ and $\lim_{x\to c} g(x) = B$, then $\lim_{x\to c} f(x)g(x) = AB$.

5. Suppose f is an odd function and is differentiable everywhere. Prove that for every positive number b, there exists a number c in $(-b,b)$ such that $f'(c) = f(b)/b$.

6. Let $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$.

(a) Find $f_x(x,y)$.

(b) Show that f is not differentiable at $(0,0)$.

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微積分試題

適用學系:數學教育學系三年級

一、填充題（40%，每格 5%）

1. Let $D_u f(p)$ be the directional derivative of f at $p=(p_1,p_2)$ in the direction $u=(u_1,u_2)$. Then the definition of $D_u f(p)$ is _______. If $f(x,y)=4x^2-xy+3y^2$, $p=(2,-1)$ and $u=(4,3)$, then $D_u f(p)=$______.

2. Evaluate $\lim\limits_{n\to\infty}\dfrac{n+\frac{n}{2}+\frac{n}{3}+\frac{n}{4}+\cdots+1}{n}=$______________.

3. Evaluate $\int_1^3\frac{1}{4+2^x}dx$ = ___________.

4. Evaluate $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{n}{k^2+n^2}=$ ___________.

5. Evaluate $\lim\limits_{n\to\infty}\left(\frac{2}{n}\right)^8\sum\limits_{k=1}^{n}k^7=$ ________.

6. The arc of the parabola $y=x^2$ from $(1,1)$ to $(2,4)$ is rotated about the y-axis. The area of the resulting surface is ________.

7. The maximum value of the function $f(x)=x^2+2y^2$ on the circle $x^2+y^2=1$ is ________.

二、計算及證明題（60%，每題 10%）

1. Let $f(x)=\begin{cases}x\sin\frac{1}{x} & \text{if } x\neq 0,\\ 0 & \text{if } x=0.\end{cases}$. Prove that f is continuous.

2. Evaluate $\int_0^\infty x^4 e^{-x} dx$.

3. Find the limit or explain that it does not exist $\lim_{x\to 0} \frac{2(x+1)^{\frac{7}{11}} - 2}{x}$.

4. Show that $\tan^{-1} x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$.

5. Suppose that f and g are continuous on $[a,b]$ and differentiable on (a,b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Prove that $f(b) < g(b)$.

6. Prove that if $a_n \geq 0$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ also converges.