國立臺中教育大學100學年度大學日間部轉學招生考試

微積分 試題

適用學系:數學教育學系　二年級

一、填充題（40%，每格5%）

1. The curve determined by $y = x^2 + 1,\ 0 \le x \le 4,$ can be put in parametric form using x as the parameter by writing $x =$ ____________ , $y =$ ____________.

2. The formula for the length L of the curve $x = f(t),\ y = g(t),\ a \le t \le b,$ is $L =$ ____________.

3. Evaluate $\lim_{n\to\infty} \dfrac{|x|^n}{n!} =$ ____________.

4. Evaluate $\lim_{x\to 0} \dfrac{\tan x}{x} =$ ____________.

5. If $x_1 = \sqrt{6}$ and $x_{n+1} = \sqrt{6 + x_n}$. Evaluate $\lim_{n\to\infty} x_n =$ ____________.

6. Evaluate $\lim_{x\to 0} (1 - 2x)^{1/x} =$ ____________.

7. Let $f'(0) = 1$. Find $\lim_{h\to 0} \dfrac{f(3h) - f(-2h)}{h} =$ ____________.

二、計算及證明題（60%，每題10%）

1. Find the volume V generated by revolving the region bounded by the curve $y = 3 - x^2$, the y-axis, and the lines $y = 1$ and $y = 2$ about the y-axis.

2. Let $f(x) = \int_x^{x^3+8} \dfrac{x}{1+\sqrt{t}}\, dt$. Find $f(1) = ?$

3. A function f is continuous on $[a, b]$ and differentiable on (a, b), and we have $f(a) = f(b)$. Prove that there exists at least one number $c \in (a, b)$, such that $f'(c) = 0$.

4. Find the area of the surface of revolution generated by revolving the curve $y = \sqrt{x},\ 0 \le x \le 4$, about the x-axis.

5. Is there a number a such that $\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exists? If so, find the value of a and the value of the limit.

6. Show that the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

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微積分 試題

適用學系：數學教育學系　三年級

一、填充題（40%，每格 5%）

1. If $\sum_{k=1}^{\infty}|a_k|$ converges, we say that the series $\sum_{k=1}^{\infty}a_k$ converges ____________.
   Give an example ____________ that $\sum_{k=1}^{\infty}|a_k|$ converges.

2. Evaluate $\int_0^{\infty}\frac{\sin x}{x}dx =$ ___________.

3. Evaluate $\lim_{n\to\infty}(\frac{n-1}{n+1})^n =$ ___________.

4. Evaluate $\int_{-a}^{a}\sqrt{a^2-x^2}\,dx =$ __________.

5. Evaluate the indefinite integral $\int e^x \sin x\, dx =$ __________.

6. If f is a continuous function such that $\int_0^x f(t)\,dt = xe^{2x} + \int_0^x e^{-t} f(t)\,dt$ for all x, then $f(x) =$ __________.

7. An equation of the tangent plane to the surface $z = y\cos(x-y)$ at the point $(2,2,2)$ is __________.

二、計算及證明題（60%，每題 10%）

1. Show that $\int_0^1 \frac{\ln x}{1-x} = \sum_{n=1}^{\infty}\frac{1}{n^2}$.

2. Find $\int_0^{16}\frac{dx}{\sqrt{9+x^2}}$.

3. Find the volume of the solid generated by revolving the region bounded by the parabolas $y = x^2$ and $y^2 = 8x$ about the x-axis.

4. Find the formula for $\sum_{j=1}^{n} (j+2)(j-5)$.

5. For what values of a and b is the following equation true?

$$\lim_{x\to 0}\left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2}\right) = 0$$

6. Let $f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$.

(1) Find $f_x(x,y)$.

(2) Show that f is not differentiable at $(0,0)$.